Electronic Design

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Gain-Bandwidth Product is Not (Always) Constant

Gain-bandwidth is always constant, isn't it? Using the inverting single-pole op amp as an example, this article explains why that often-held belief is a fallacy.

When talking amplifiers, I sometimes get the impression that whole groups of electronics hobbyists and engineering students have been brainwashed by endlessly hearing and repeating the mantra "Gain-bandwidth product is a constant." Hearing this statement so often, they actually start to believe this is a universal truth.

I was reminded of this once again some time ago, when I saw a video on the otherwise excellent YouTube channel "w2aew" The video has been titled "#172: Basics of Op Amp Gain Bandwidth Product and Slew Rate Limit"[1]. Around minute 2:23, the host says "...for a single-pole response, the product of the DC-gain and the bandwidth is constant." However, this statement isn't universally true—sometimes it's true, sometimes it's approximately true, and sometimes it's completely wrong.

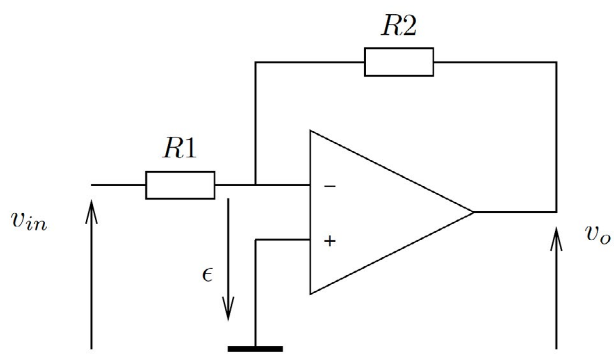
You might have thought I was going to tell you about the case of a voltage amplifier with a current feedback op amp—a statement that you probably already knew isn't true—but I am not. Instead I'll focus on what's probably the simplest op-amp voltage amplifier configuration you can imagine, the mother of all op-amp circuits: the inverting single-pole op-amp amplifier shown in *Figure 1*.

To avoid confusion, let's first define all symbols that will be involved *(see table)*. To deduce the connection between $A_{OL,0}$, BW_{OL}, $A_{CL,0}$, and BW_{CL}, all we need to do is apply Kirchhoff's law to the inverting node of the op amp. We assume our op amp is or can be approximated by a single-pole model, which means we can write:

$$\overline{A}_{OL} = \frac{A_{OL,0}}{1+j\frac{f}{BW_{OL}}} \qquad (1)$$

Applying Kirchhoff's law gives:

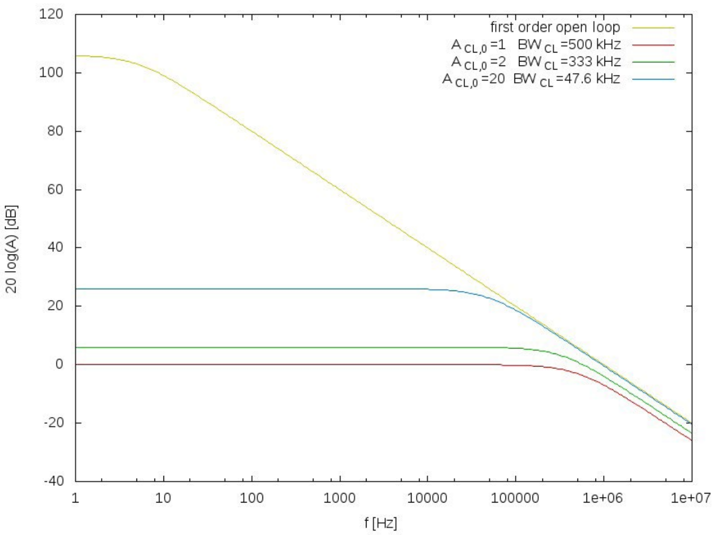
$$\frac{\overline{v}_{in}+\overline{\epsilon}}{R1} + \frac{\overline{v}_o+\overline{\epsilon}}{R2} = 0$$



1. Standard inverting op-amp amplifier.

Symbols Used in Article

Symbol	Definition
$\overline{A}_{CL}$	Small signal closed-loop voltage gain: $\frac{\overline{v}_o}{\overline{v}_{in}}$
A_{CL}	Magnitude of small signal closed-loop voltage gain: $\lvert\overline{A}_{CL}\rvert$
$A_{CL,0}$	Absolute value of DC-closed-loop voltage gain
$\overline{\epsilon}$	Voltage difference signal at op-amp input terminals
$\overline{A}_{OL}$	Small signal open-loop voltage gain: $\frac{\overline{v}_o}{\overline{\epsilon}}$
A_{OL}	Magnitude of small signal open-loop voltage gain: $\lvert\overline{A}_{OL}\rvert$
$A_{OL,0}$	DC-open-loop voltage gain
BW_{OL}	Open-loop bandwidth
BW_{CL}	Closed-loop bandwidth



2. Amplitude characteristics for a standard inverting op-amp amplifier.

We also have:

$$\overline{v_o} = \overline{A}_{OL} \cdot \overline{\epsilon}$$

which gives us:

$$\frac{\overline{v}_o}{\overline{v}_{in}} = -\frac{R2\overline{A}_{OL}}{R2+R1\overline{A}_{OL}+R1} \qquad (2)$$

Substituting Equation 1 in Equation 2 gives:

$$\overline{A}_{CL} = -\frac{\frac{A_{OL,0}\cdot R2}{R1+R2+A_{OL,0}\cdot R1}}{1+j\cdot\frac{f}{\frac{BW_{OL}\cdot(R1+R2+A_{OL,0}\cdot R1)}{R1+R2}}} \qquad (3)$$

Equation 3 might look a bit daunting at first sight, but it was intentionally written that way to allow identification with a single-pole representation of the system:

$$\overline{A}_{CL} = -\frac{A_{CL,0}}{1+j\cdot\frac{f}{BW_{CL}}} \qquad (4)$$

Inspecting Equations 3 and 4, we get:

$$\begin{cases} BW_{CL} = BW_{OL}\cdot\frac{R1+R2+A_{OL,0}\cdot R1}{R1+R2} \\ A_{CL,0} = \frac{A_{OL,0}\cdot R2}{R1+R2+A_{OL,0}\cdot R1} \end{cases} \qquad (5)$$

Multiplying the respective left and right sides in Equation 5 gives us the following exact formula:

$$A_{CL,0}\cdot BW_{CL} = A_{OL,0}\cdot BW_{OL}\cdot\frac{R2}{R1+R2} \qquad (6)$$

Equation 6 is seldom used because in most practical cases:

$$A_{OL,0} \gg 1+\frac{R2}{R1}$$

which implies:

$$A_{CL,0} \approx \frac{R2}{R1}$$

and therefore the equation can be rewritten as:

$$\left(1+\frac{R2}{R1}\right)\cdot BW_{CL} = A_{OL,0}\cdot BW_{OL}$$

$$\Leftrightarrow \left(1+A_{CL,0}\right)\cdot BW_{CL} = A_{OL,0}\cdot BW_{OL} \qquad (7)$$

A closer look at Equation 7 shows the cause of the problem with the mantra: Only when $A_{CL,0}$ is substantially greater than 1 can that equation be approximated by $A_{CL,0}\cdot BW_{CL} = A_{OL,0}\cdot BW_{OL}$ for the standard inverting op-amp amplifier. If this isn't the case, though, we can get a substantial error.

Take, for example, the case of a phase inverter (i.e., $R1 = R2$ or $A_{CL,0} = 1$) with a 741 op amp, for which $A_{OL,0} = 200000$ and $BW_{OL} = 5$ Hz. Using the mantra, we would find a closed-loop bandwidth of 1 MHz. However, using Equation 7, we find the correct bandwidth to be 500 kHz, which means the mantra gives us an overestimation of the bandwidth with a factor 2! More generally, the procentual error on the predicted closed-loop bandwidth using the mantra-formula is given by:

$$\Delta\% = \frac{1}{A_{CL,0}}\cdot 100\%$$

So, if you want your prediction to be less than 5% off, your closed-loop DC gain should be at least 20 i.e. 26dB.

The graphs in *Figure 2*, which were made using PSpice and a first order op-amp model, also show $A_{CL,0} \bullet BW_{CL} \neq A_{OL,o} \bullet BW_{OL}$, since the oblique asymptotes of the closed-loop system don't coincide with the one of the open-loop system as is often erroneously drawn in textbooks.[2] To clarify this point, consider:

$$A_{CL} = \frac{A_{CL,0}}{\sqrt{1+\left(\frac{f}{BW_{CL}}\right)^2}}$$

For $f \to \infty$, this becomes:

$$A_{CL} = \frac{A_{CL,0}BW_{CL}}{f}$$

Considering the vertical axis in *Figure 2* is in dB and the horizontal axis is logarithmic, we get:

$20\log(A_{CL}) = 20\log(A_{CL,0}BW_{CL}) - 20\log f$. The term $20\log(A_{CL,0}BW_{CL})$ determines the vertical position of the oblique asymptotes. Because of Equation 7, the latter term is equal to:

$$20\log\left(\frac{A_{CL,0}}{1+A_{CL,0}}A_{OL,0}BW_{OL}\right)$$

and obviously a function of $A_{CL,0}$ itself. Only if $A_{CL,0} >> 1$, the oblique asymptote of $20\log A_{CL}$ will coincide with the one of $20\log A_{OL}$.

As an afterthought, note that for a standard *non*-inverting op-amp amplifier, the mantra-formula is *correct*, but for the standard inverting configuration you'd better memorize $(1 + ACL,0)BWCL = AOL,0\ BWOL$ from now on.

When actually measuring the bandwidth of the above-mentioned unity-gain inverter, don't be surprised if you notice some peaking in the amplitude characteristic. This is caused by a complex pole in the real 741 and the presence of parasitic capacitance between the inverting and non-inverting input of the op amp, but that's another story.

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References:

1. https://www.youtube.com/watch?v=UooUGC7tNRg
2. *Analog Design Essentials* by Willy M. C. Sansen, p.155

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