Determine Equivalent ESR, Ripple Voltage, And Currents For Unequal Capacitors In Parallel

ALEXANDER ASINOVSKI | MURATA POWER SOLUTIONS, MANSFIELD, MASS. aasinovski@murata.com

CAPACITORS OFTEN ARE CONNECTED in parallel in power electronics to decrease high-frequency ripples, current stress, power dissipation, and operating temperature, as well as to shape frequency response and boost reliability. Yet designers have three critical questions about this technique:

- What are the equivalent values of capacitance C_{se} and equivalent series resistance (ESR) R_{se}?
- What is the high-frequency ripple voltage?
- What are the individual RMS currents?

If all N capacitors in the parallel connection are identical *(Fig. 1)*, with equal capacitance values $C_{sk} = C$ and equal ESR values $R_{sk} = R_s$, then for k = 1, 2, . . . N the answers are clear:

- C_{se} is directly proportional to the number of capacitors N: $C_{se} = NC$, and R_{se} is inversely proportional to N: $R_{se} = R_s/N$.
- Ripple voltage V (RMS value) is:

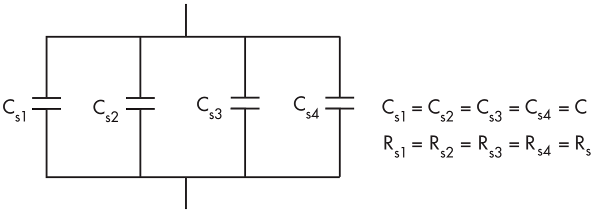
$$V = I\sqrt{R_{se}^2 + X_{se}^2} \qquad (1)$$

for a sinusoidal current excitation $i(t) = I\sqrt{2}\sin(2\pi ft)$ with frequency f, where $X_{se} = 1/(2\pi fC_{se})$ is the reactance of the equivalent capacitor C_{se} and RMS value I, and individual RMS currents in the capacitors are identical: $I_k = I/N$.

When the capacitors in the parallel connection aren't identical, with different capacitance C_{sk} and ESR R_{sk} values, the solution to the problem isn't trivial. The direct approach is to obtain an analytical expression for the input impedance of the parallel connection in the algebraic form $Z = \mathrm{Re}\,Z - j\,\mathrm{Im}Z = Z_{se}$ Z and use the formulas $R_{se} = \mathrm{Re}\,Z$, $X_{se} = \mathrm{Im}\,Z$, and $C_{se} = 1/(2\pi f\,X_{se})$.

A less complicated approach is based on the conversion of series C_{sk}, R_{sk} connections to equivalent parallel C_{pk}, R_{pk} connections. To obtain relationships between R_{pk} and R_{sk}, and also between C_{pk} and C_{sk}, set the admittance Y_{pk} of the parallel C_{pk}, R_{pk} pair and admittance Y_{sk} of the series C_{sk}, R_{sk} pair connections equal to each other: $Y_{pk} = Y_{sk}$, $\mathrm{Re}(Y_{pk}) = \mathrm{Re}(Y_{sk})$, and $\mathrm{Im}(Y_{pk}) = \mathrm{Im}(Y_{sk})$. Then:

$$C_{pk} = \frac{C_{sk}}{1+\left(\frac{R_{sk}}{X_{sk}}\right)^2} \qquad (2)$$



1. For an array of N identical capacitors in parallel, determining the total equivalent capacitance and ESR values is straightforward. For unequal capacitors, the calculation can be difficult.

$$R_{pk} = \frac{R_{sk}^2 + X_{sk}^2}{R_{sk}} \qquad (3)$$

where:

$$X_{sk} = \frac{1}{2\pi fC_{sk}} \qquad (4)$$

is the reactance of the individual capacitor.

After individual parallel capacitances C_{pk} and resistances R_{pk} are calculated according to Equations 2 and 3, equivalent parallel capacitance C_{pe} can be easily found as the sum of C_{pk}:

$$C_{pe} = \sum_{k=1}^{N} C_{pk} \qquad (5)$$

The real part of equivalent admittance can be found as the sum of admittances $1/R_{pk}$. R_{pe} can be obtained as a reverse value of that sum:

$$R_{pe} = \frac{1}{\sum_{k=1}^{N}\frac{1}{R_{pk}}} \qquad (6)$$

The system's equivalent series capacitance C_{se} and ESR R_{se} can be found by conversion of the parallel C_{pe}, R_{pe} connection to the equivalent series connection C_{se}, R_{se}. To obtain relationships between C_{se} and C_{pe} and also between R_{se} and R_{pe}, set impedance Z_{pe} of the parallel C_{pe}, R_{pe} and impedance Z_{se} of the series C_{se}, R_{se} connections equal to each other: $Z_{pe} = Z_{se}$, $\mathrm{Re}\,Z_{pe} = \mathrm{Re}\,Z_{se}$, $\mathrm{Im}\,Z_{pe} = \mathrm{Im}\,Z_{se}$. Then:

$$C_{se} = C_{pe}\left[1+\left(\frac{X_{pe}}{R_{pe}}\right)^2\right] \qquad (7)$$

$$R_{se} = \frac{R_{pe}}{\left[1 + \left(\frac{R_{pe}}{X_{pe}}\right)^2\right]} \qquad (8)$$

where:

$$X_{pe} = \frac{1}{2\pi f C_{pe}} \qquad (9)$$

is the reactance of the equivalent parallel capacitor C_{pe} *(Equation 5)*.

Based on this analysis, the calculation procedure for equivalent series capacitance C_{se}, ESR R_{se}, voltage ripples V, and RMS currents I_k in the capacitors is:

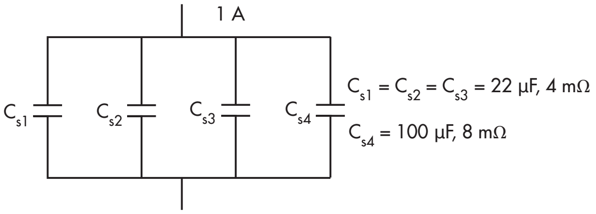
- Calculate reactances of individual capacitances according to Equation 4.
- Determine equivalent parallel parameters C_{pk}, R_{pk} of the capacitors based on Equations 2 and 3.
- Calculate equivalent parallel capacitance C_{pe} of the structure, its reactance X_{pe}, and equivalent parallel resistance R_{pe} according to Equations 5, 9, and 6.
- Calculate equivalent series capacitance C_{se} and ESR R_{se} of the structure according to Equations 7 and 8.
- Obtain RMS ripple voltage V using Equation 1.
- Calculate RMS currents I_k in the capacitors based on:

$$I_k = \frac{V}{\sqrt{R_{sk}^2 + X_{sk}^2}} \qquad (10)$$

Note that ESR values R_{sk} are strong functions of frequency. A designer should use ESR data specified by capacitor manufacturers at a given frequency of operation, such as the data for ceramic and polymer aluminum electrolytic capacitors from Murata Manufacturing Co. Ltd. (MMC) *(http://ds.murata.co.jp/software/simsurfing/en-us/index.html)*.

To illustrate the calculation procedure, let's determine equivalent parameters, voltage ripple, and current distribution for a parallel connection of three ceramic capacitors (GRM-21BR60J226ME39L) and one polymer capacitor (ESASD-40J107M015K00) from MMC *(Fig. 2)*. Using the data f = 200 kHz, $C_{s1} = C_{s2} = C_{s3} = 22$ µF, $R_{s1} = R_{s2} = R_{s3} = 4$ mΩ, $C_{s4} = 100$ µF, $R_{s4} = 8$ mΩ, I = 2 A, then:

- For reactance of each individual capacitance according to Equation 4, we have $X_{si} = X_{s2} = X_{s3} = 3.6$ mΩ, $X_{s4} = 0.8$ mΩ.
- Equivalent parallel parameters C_{pk}, R_{pk} of the capacitors based on Equations 2 and 3 are $C_{p1} = C_{p2} = C_{p3} = 21.7$ µF, $R_{p1} = R_{p2} = R_{p3} = 331$ mΩ, $C_{p4} = 49.7$ µF, $R_{p4} = 16$ mΩ.



2. This example of four capacitors, with three identical and one different, illustrates how the computation scheme works in practice.

- For equivalent parallel capacitance C_{pe}, its reactance X_{pe} and equivalent parallel resistance R_{pe} of the structure according to Equations 5, 9, and 6, we calculate $C_{pe} = 115$ µF, $X_{pe} = 6.9$ mΩ, $R_{pe} = 13.9$ mΩ.
- According to Equations 7 and 8, the equivalent series capacitance C_{se} and ESR R_{se} are $C_{se} = 143.4$ µF, $R_{se} = 2.76$ mΩ.
- For RMS ripple voltage V based on Equation 1, we obtain V = 12.4 mV.
- RMS currents according to Equation 10 in ceramic and polymer capacitors are respectively: $I_1 = I_2 = I_3 = 341$ mA, $I_4 = 1.1$ A.

This shows the technique can easily determine the parameter values in each of the capacitors.

ALEXANDER ASINOVSKI is principal engineer at Murata Power Solutions Inc., Mansfield, Mass. He holds BSEE and MSEE degrees from State Technical University, St. Petersburg, Russia, and a PhD from the University of Telecommunications, St. Petersburg.